CMC 2020 Grading Guide

Instructions for leaders

First of all, thank you for your work in making CMC possible. The grading of the CMC is a collective process where the leaders and coordinators will work together to determine a score and feedback for each submission.

AoPS leaders forum Please regularly check the AoPS leaders forum[1] for discussions and updates, e.g., amendments/clarifications regarding the grading scheme. All leaders and coordinators have access to this forum.

Rubric The coordination team has put together a detailed grading rubric for each problem, presented in this document.

If there are any general-interest questions, please ask them in the AoPS leaders forum, and the coordination team will be happy to clarify any issues.

Questions specific to a submission should be asked in the discussion thread attached to the submission in the AoPS grading site[2].

There are "general rules" in the grading scheme on how to apply the rubric for each problem. These rules may differ from problem to problem. Some rubric items are "additive", meaning that points may be combined, while some other rubric items are not additive, meaning that one should take the maximum among options.

Score and feedback The AoPS grading site asks for a feedback text in addition to a numerical score recommendation for each submission. For partial credit or deductions, it would be helpful to the coordination team if the leader can refer to specific items in the rubric (e.g., "`2 points due to rubric item (3a)`"). Other comments such as "`similar to official solution #2`" may be helpful.

You may use the discussion thread for additional comments that may be helpful to the coordination team (these discussions will not be sent to contestants).

At the conclusion of the exam, all numerical scores will be made public, while the feedback texts will be sent to individual contestants.

General team comments for each problem Leaders, please post a brief general comment about your team's work on each problem in the discussion thread of 01-numbered contestant.

E.g., the Colombian leader can leave a comment in the Problem 3 discussion thread of COL01 something like "`only COL03 solved this problem, while several others (COL01, COL04, COL05) observed that ...`". These general comments will help the coordination team to quickly assess the recommended scores.

Discussions The coordination team may flag (dispute) some submission for additional discussions. In this case, a coordinator will leave a comment in the discussion thread attached to each submission. Leaders, please check the disputed entries and work with the coordinators and edit your score/feedback if necessary to arrive at an agreement. In case of unresolved disputes, the problem captains have the final say in setting the score/feedback.

A trust-based process We rely on the leaders to do the bulk of the work in grading the exams, with the help of the coordination team to ensure that the rubric is applied fairly to all participants. Due to time and workload constraints, the coordination team will be expected to check the leaders' grading only selectively (unlike, say, during IMO coordination).

An open and transparent platform All coordinators and leaders can view all submissions and discussions on the AoPS grading site, and may comment on any entry (though leaders may only grade their own team's submissions). We hope that this open platform will aid in a fair grading system. It would helpful if you can identify your role (e.g., COL leader) while posting in the discussions.

[1] https://artofproblemsolving.com/community/c1221109_cmc_forum_for_leaders
[2] https://artofproblemsolving.com/contests/cmc/grader

Problem 1 grading scheme

General rules for this problem

- Item 1 cannot be combined with any other item, it applies only if it is the only point given. Items 2 through 5 are additive.
- Sub-items within item 5 are not additive.
- Minor math mistakes generally lead to the deduction of 1 point.
- Any equivalent approach should receive proper and proportionately judged equivalent marks.

Rubric

1. Answer, correct and clearly written down, with little or no justification. .... **1 point**
2. Proving that for $n \equiv 2 \pmod 3$ no tiling is possible .... **1 point**
3. Proving no tiling exists for $n = 4$ .... **0 points**
4. Proving no tiling exists for $n = 6$ .... **1 point**
5. Construction of the solutions (for $n \equiv 0, 1 \pmod 3$ and $n \neq 4, 6$) .... **5 points**
   - (5a) Proving a tiling exists for $n = 3, 7$ or 9 .... *0 points*
   - (5b) Proving a tiling exists for some even $n \geq 10$ .... *1 point*
   - (5c) Proving that there is a some congruence class and an integer k such that, if n is in this congruence class and has a solution then so doea $n + k$ .... *1 point*
   - (5d) Completing both (5b) and (5c) .... *2 points*
   - (5e) An inductive or iterative argument that reduces the construction to finitely many cases, but is missing one or more base cases .... *4 points*
   - (5f) A correct construction with all necessary base cases .... *5 points*

Problem 2 grading scheme

General rules for this problem

- We expect solutions using induction, in which one shows that $f(n)$ is divisible by at most 1 *new prime* (i.e., a prime not dividing any of $f(1), \ldots, f(n-1)$).
- Top-level numbered items are not additive. Subitems in **2.** are additive.
- Any equivalent approach should receive proper and proportionately judged equivalent marks.

Rubric

1. Observations with zero credit .... **0 point**

- (1a) Verifying the result for specific values of n .... *0 points*
- (1b) Proving general divisibility results for $f(n)$, such as $4 \mid f(n)$ for odd n, or $7 \mid f(n)$ for $n \equiv 3 \mod 7$ .... *0 points*
- (1c) Proving $p \neq n$ .... *0 points*

2. Partial credit (additive) .... **3 point**

- (2a) Starting the inductive approach: stating/conjecturing there is ≤ 1 new prime (whether or not induction is explicitly mentioned) .... *1 point*
- (2b) Proving that there are no new primes $p < n$ .... *1 point*
- (2c) Proving that there are no new primes $n < p < 2n$ .... *1 point*

3. All three claims in **2.** stated, two claims proved, and otherwise a complete solution. .... **5 points**

4. Minor error (rule of thumb: obvious to student once pointed out, less than one sentence and two minutes to fix) .... **-1 points**

5. Complete and correct solution .... **7 points**

Problem 3 grading scheme

General rules for this problem

- We expect solutions to use either a synthetic or computational approach. Points from the two approaches cannot be added.
- In the synthetic approach, items **2.** and **3.** can be combined, for a maximum of 3 points partial credit. Note that the subitems under **2.** and **3.** are illustrative examples only; it's possible for other statements to meet the general criterion, and those statements get points as well.
- Do not deduct points for configuration issues.
- Minor math mistakes generally lead to the deduction of 1 point, but should be dealt with on a case-by-case basis.
- Any equivalent approach should receive proper and proportionately judged equivalent marks.

Rubric for a synthetic approach

1. Any statement not covered under items **2.** or **3.** below. .... **0 points**

(1a) Finding that the desired point must lie on the line BC. .... *0 points*

(1b) Noting that BI and EJ intersect at M, the midpoint of the arc $\widehat{AC}$. .... *0 points*

(1c) Noting that $AM = JM = CM$. .... *0 points*

(1d) If $N = EI \cap CJ$, concluding that $MNIJ$ is cyclic. .... *0 points*

(1e) Noting (spiral) similarity between triangles BAD and EAC. .... *0 points*

(1f) Noting (spiral) similarity between triangles AIB and AJE. .... *0 points*

2. Identifying the intersection point K, in a way that is evidently independent of D. (No points for choosing a particular D_0 and intersecting the resulting line I_0J_0 with BC.) .... **1 point**

(2a) Define K as the point on BC with $BK = BA$. .... *1 point*

(2b) Define K as the point on BC with $MK = MA$. .... *1 point*

3. Any statement that allows one to identify the direction of line IJ or JK, regardless of whether K is defined as the intersection of IJ and BC, or by one of the two characterizations in **2.**. (No points for determining the direction of IK, unless it is shown that this line coincides with IJ.) ...... **2 points**

(3a) Concluding that at least one of the quadrilaterals $AIJM$, $AJCK$ is cyclic. .... *2 points*

(3b) Proving similarity between AIJ and ABE (or between AIJ and ADC). .... *2 points*

(3c) Applying Pascal's Theorem to get $IJ \parallel BN \parallel PE$. .... *2 points*

4. Complete solution. .... **7 points**

Rubric for a computational approach

1. Any partial progress other than the items below (unless intermediate results are interpreted synthetically, in which case, see the synthetic rubric). .... **0 points**

2. Writing, without proof, a correct formula for the point K only depending on A, B, C. ...... **1 point**

3. Complete solution with no mistakes, making clear that the student performed every step of the calculation. .... **7 points**

Problem 4 grading scheme

General rules for this problem

- The top numerical items are not additive.
- Grading should generally be 0 or 7. Significant progress may yield partial credit as determined on a case-by-case basis — we expect this to be rare.
- Minor math mistakes generally lead to the deduction of 0 or 1 point, but should be dealt with on a case-by-case basis.
- Any equivalent approach should receive proper and proportionately judged equivalent marks.

Rubric

1. Reformulations .... **0 points**

- (1a) Converting the board to a graph .... *0 points*
- (1b) Reducing the problem to bounding length of a "snake-in-a-box" path .... *0 points*

2. Easy bounds and ideas .... **0 points**

- (2a) Showing an $n^2/2 + O(n)$ bound .... *0 point*
  A typical example of this would be noticing that among any 2×2 subgrid, at most two squares can be on the king path.
- (2b) Considering squares with even row/column (i.e. special vertices in the solution) .... *0 points*
- (2c) Bounding the number of moves/squares in a fixed-size subregion (e.g. 2×2 or 3×3) ... *0 points*

3. Approach considering a generalization of the statement to a disjoint union of "snake-in-a-box" paths (i.e., allowing the king to jump arbitrarily) .... **1 point**

- (3a) Correctly stating that there are $\leq (n^2 - 1)/2$ king-moves in this generalization. .... *1 point*

4. Showing an $n^2/2 + O(1)$ bound .... **5 points**

This is likely a result of slightly suboptimal use of blocks and covering.

5. Correct solution .... **7 points**

Problem 5 grading scheme

General rules for this problem

- The points from the two approaches (item **1.** and **2.**) are not additive, so take the maximum from the two items.
- Item **3.** is additive with the previous two items.
- The sub-items within each item are additive.
- Explicit numerical examples (e.g. $3 + (-5)$ instead of $a + b$) are acceptable substitutes of algebraic expressions, as long as they do not introduce possible confusions (which could happen when the magnitude of the two numbers are equal or in a ratio of $1 : 2$).
- Minor math mistakes (aside from the ones in item **3.**) generally lead to the deduction of 1 point.
- Any equivalent approach should receive proper and proportionately judged equivalent marks.

Rubric

1. Induction approach (solution 1) . . . . **7 points**

(1a) Statement of the lemma . . . . *+2 points*
Any statement to the effect of "for every state Zuming can reach, he can also reach the absolute-valued version" is acceptable.

(1b) Proving the lemma . . . . *+5 points*
This of course depends getting points on the previous subitem.

i. Forgetting the base case . . . . *-0 points*

2. Construction approach (solution 2) . . . . **7 points**

(2a) Showing that there is at least one subtraction step *and* flipping one of them . . . . *+1 point*
This point can be awarded even they didn't flip the last one (chronologically if we consider the moves as a sequence, or topologically if we consider them as a computation tree).

(2b) Making a correct claim about propagating the sign flip from the previous subitem to the last number . . . . *+1 point*

(2c) Justifying the propagation of the sign flip to the last number . . . . *+5 points*
The latter two subitems can be awarded without the first subitem if e.g. they forgot to argue that there is at least one subtraction.

i. Failure to handle later subtractions due to not explicitly flipping the last one . . . . . . *-2 points*
Note that this also entails not getting the point in (2b).

3. General deductions

This item only applies if the solution is otherwise mostly correct (that is, the solution has got points from subitem (1b) or (2c)).

(3a) Neglecting to mention that multiplication/division preserves sign flip/magnitude. . . . . *-1 point*

(3b) Minor sign errors for addition/subtraction cases . . . . *-1 point per error, -2 points max*

Problem 6 grading scheme

General rules for this problem

- In what follows, the numbered items are additive but the sub-items (e.g. 2a) are not.
- Minor math mistakes generally lead to the deduction of 1 point.
- Do not deduct points for poor or missing diagrams.
- Any equivalent approach should receive proper and proportionately judged equivalent marks.

Rubric

1. Answer only .... **0 points**

2. Complete construction for odd n .... **2 points**

In general, if a construction works as written, then full points should be awarded, even if an explanation or justification for why the construction exactly works is omitted.

(2a) Construction which works for all but finitely many n .... *2 points*

In other words, if the student makes a mistake which only affects a finite set of n, then we do not deduct any points.

(2b) Construction which works, except leads to non-convexity for infinitely many n .... *1 point*

This only applies to constructions which actually fail convexity as written. If the student's construction is valid and the only issue is that they do not verbally justify the convexity of their resulting polygon, then the student should earn full marks.

(2c) Construction which works perfectly for infinitely many n .... *1 point*

This point is not awarded if there is some also some issue as in (2b).

(2d) Construction which works for only finitely many n .... *0 points*

In other words, special cases are not worth any marks.

3. Proof that for even n, the polygon is actually regular .... **5 points**

(3a) Explicitly stated idea to treat the sides of the polygon as vectors (or complex numbers) which sum to zero .... *1 point*

The student doesn't need to explicitly mention the sum is zero.

(3b) Reflecting the polygon over the distinguished angle or side .... *1 point*

See the third solution for a solution using this idea.

(3c) Manages to either sum or cancel the vectors (or complex numbers), in such a way as to obtain a "closed form" or sum with $O(1)$ terms, for example as in either official solution .... *3 points*

This item can be obtained even if the calculation is done on a single projection, e.g. if a student has projected onto the y-axis as in the first solution, and cancelled most of the vectors. Basically, one can think of this item as any result derived from the observation that "adding equally spaced vectors is easy".

On the other hand, to earn this item, the sum must actually be completed, i.e. no points are awarded for merely the hope that the sum should simplify, the implementation needs to be finished.

(3d) Managing to show that the polygon is equilateral .... *4 points*

(3e) Managing to show that the polygon is equiangular .... *4 points*

(3f) Proving the result only for certain n, such as a proof for $n = 4$ .... *0 points*

(3g) Proving the result for all even n with the extra assumption that the special angle and special side are adjacent, farthest away, etc. .... *1 point*

Problem 7 grading scheme

General rules for this problem

- The top-level numbered items are additive. Within item 2, take the maximum that applies.
- Any equivalent approach should receive proper and proportionately judged equivalent marks. For novel approaches, claims for partial credit should be aligned with this scheme, relative to a complete solution (e.g., one appearing in a public or private AoPS post).

Rubric

1. Any construction achieving the correct answer .... **1 point**

- No credit for a correct answer without a construction. Conversely, no deduction for an incorrect or missing answer if the construction is correct.
- No extra credit for describing all maximal configurations.

2. Proof of the upper bound .... **6 points**

(2a) Reformulation, such as: .... *0 points*

- Statement that in an optimal configuration, changing any one cell cannot increase the sum.
- Rewriting the sum in terms of entries of a $(0, 1)$-matrix.
- Sorting the rows or columns, but not both.
- Proposing induction on n.

(2b) Nontrivial pertinent observation, such as: .... *1 point*

- Observation that changing cell (i, j) from white to black changes the sum by $a_i - b_j$.
- Statement of the inequality $a_1 + b_1 + \cdots + a_i + b_i \leq 2ni - i^2$.
- Introduction of a second objective function that occurs as part of some complete proof (e.g., $\sum_{i=1}^{n} a_i^2$).
- Use of the rearrangement inequality to sort both the rows and the columns.

(2c) Substantial partial result or construction, such as: .... *3 points*

- Definition of the sets A, B, C from Solution 1.
- Regrouping a sum of the form $\sum_{i,j,k} x_{ij}(1 - x_{ki})$ into cyclic orbits (see Solution 2).
- Reduction of the original problem to the echelonized case (see Solution 3).
- Solving the echelonized case (see Solution 3).
- Statement and application of the claim from Solution 5.
- Proof that if $i \neq j$, that (i, j) and (j, i) must be opposite colors (see Solution 6).

(2d) An otherwise complete proof with a minor mistake .... *5 points*

- In Solution 5, missing the case where j is undefined.

(2e) Complete proof .... *6 points*

Problem 8 grading scheme

General rules for this problem

- Take the maximum of all items which apply.
- Aside from the specifically listed partial credit, grading should generally be 0 or 7.
- Minor math mistakes generally lead to the deduction of 1 point.
- Any equivalent approach should receive proper and proportionately judged equivalent marks.

Rubric

1. Reformulations and intermediate bounds. .... **0 points**

- (1a) Proof that S_n or Q_n are decreasing. .... *0 points*
- (1b) Prove that S_n or Q_n have limiting values. .... *0 points*
- (1c) Proof of any bound of the form $Q_n^2 - Q_{n+1}^2 \geq \frac{C}{n^2}$ for large enough n, unless this bound implies item 2 below. .... *0 points*
- (1d) Attempts to bound $\sum_{1 \leq i < j \leq n} (a_i - a_j)^2$ that do not relate it to a telescoping sum involving $Q_n^2 - Q_{n+1}^2$. .... *0 points*

2. Proof that if $a_1, \ldots, a_k$ are not all equal there exists $C > 0$ independent of n so that $Q_n^2 - Q_{n+1}^2 \geq \frac{C}{n}$ for $n \geq k$. .... **3 points**

3. Correct solution. .... **7 points**